Web-Based Radio Show

Series on Learning Differences, Learning Challenges, and Learning Strengths:

Mathematical Thinking and Reasoning Mathematical Thinking and Reasoning

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Good morning. This is Dr. Greenspan coming to you via our web-based radio show. Today we're continuing our series on learning: learning differences, learning challenges, and learning strengths. We've been talking about the metaphor of the tree with the branches, the root system, and the tree trunk. Today we're continuing with the branches and we're going to add to what we talked about last time when we talked about reading and reading comprehension and oral and written expression. Today we're going to talk about math and mathematical thinking and reasoning.

Remember our basic principle: Optimal learning for all children, especially children with learning challenges and individual differences in the way they process information, involves using all the senses and, as much as possible, the motor system and as much high-level thinking as possible, i.e., our tree trunk. Most importantly, we also want to have strong affect and strong emotional investment on the part of the child, so what we do has to be emotionally meaningful. Nowhere is this more important than when we consider math.

Math builds heavily on what we talked about in the root system, that is, on our visual-spatial processing and thinking, because it involves perceiving and understanding quantity, which exists in space, and also telling time, which exists into the dimension of time, as well. These are continuous phenomena – when you go from one to ten or from a little to a lot, you're really talking about something that is continuous. It's not like a word that has a specific meaning, like "good" vs. "bad." Nor is it like "more" or "less" where you can have a little bit more, a little bit less, a little, little bit more, a little, little bit less, etc. So you can draw a line and at any point in that line you can use it to illustrate a little, or a little more, or a little more, or a little more. So it's easier when you

can picture these things in space. Even time can kind of be pictured, like we do when we look at a clock. When we see the moving hand we see how much time has progressed, although we have a "feel" for time, as well. But math is very experientially based, and I always like to use the example that a young child, before he learns numbers or how to add or subtract, is already using his emotions to get a sense of quantity or mathematical thinking because a "lot" to a two-year old is a little more than you want or expect and a "little" is less than you want or expect – so it's relative to your emotional expectations. When you see a big cookie or a little cookie, you might be more attracted to the big one and that's how you develop your sense of, "Oh that's a big one," or because you're comparing it to the little one automatically and you're making a preference for the big one. As you get a little older the same thing may occur with a glass of juice or a soda.

Now how do we turn this into the best approach to master math? Well, first let's talk about the most common mistake we make. If we approach math from the point of view of just memorizing math facts, like 2+2=4, 4+4=8, adding, subtracting, or multiplication tables, you may have a child with a good memory but who doesn't have a what I call a "feel" for math. In other words, he doesn't have a sense of quantity. Then it's hard to go beyond just those memorized facts and figure things out. If you want to memorize facts they should just be icing on the cake after the child understands the basic concepts involved.

How can we teach those basic concepts? Again, let's come back to our multi-sensory, emotionally invested, motor-based experiences. We can start with preschoolers or with early school-age children who haven't had this experience in their preschool years, but we always start where the child is because we see many children with learning challenges and learning differences who, for a variety of reasons, haven't had mastery of these experiences. Sometimes their own biology makes it harder to master, so they haven't mastered it during the preschool years and they need to work on this during the school years. Step one is to master the basic concepts involved in addition, subtraction, and even multiplication and division, and even percentages and decimals. We can do that all together, as remarkable as that sounds.

So you can cover the first four years of school in the same kinds of experiences. Here's how we do it. We find things that a child is really interested in – be it clay, be it cookies, be it blocks, be it little cars – whatever the child's passion. Let's say starting at age three and a half or four, but again it could be with an older child, step one is we would simply work on "more" versus "less" or "big" versus "little." So we might have

cookies in a line and we might have a big line of four cookies and a little line with two cookies and we ask which one the child wants and which one is longer or which one is "big" and which one is "little" and have him point. Then we can put the cookies in a stack – a tower – and have him do the same thing with the cookies or with his cars or with some other favorite thing of his – a toy or a goodie.

Here we're showing him two things – we're showing him the differences between "little" and "big" or "a lot" and "a little" but we're also showing him it can exist in many different forms, including a tower form that looks like a train. This is also his first lesson in conservation, showing him that quantities exist independently of the form of their presentation. We want to work on those things and let him experiment. Does he like the objects better in a train form or the tower form? Let him stack them up into a tower or lay them out like a train, and then we can make a little game out of it. He has to decide which has more, the train or tower that has two or the one with four; and then we have a contest to see whether we can put these in different forms, like a tower or a train or a circle or a square. In other words, we see what kind of different shapes we can make with it. We can do it a couple of times and let him copy us, and then the child can do it. So, with a four or five-year old, a child who is in late preschool, it should be a piece of cake. For children who have delays or challenges, as they become more verbal they will participate in these interactions and they can get there, too.

In general, children won't be able to fully participate in these exercises until they have some degree of mastery of building bridges between ideas, level six in our tree trunk, where they can connect ideas together and answer basic "why" questions, like "Why do you want to go outside?" "Because I want to play" or "Because it'll make me happy." Then you can have the discussions necessary to do this. Before that, the child can still show preferences, but at the toddler level, i.e., an 18-month old will prefer more rather than less of something he really likes. So you can begin the preliminary stages of this with just preferences for more or less, bigger versus littler even beforehand and also show him his preferences in different forms or shapes, such as the tower or the train, to get him ready. But the real participation will occur once the child is at least answering "why" questions, which will ordinarily occur between the ages of three and three and a half or three and four years old.

So that is step one. Then, once the child has a sense of quantity in different forms and shapes and just basic things for blocks or cookies or cars, then we go to step two. Step two involves adding or subtracting from what we have. So we'll have a train of

five blocks on the floor and we'll add on four more and compare it to the second train and ask, "Which one is bigger now? Oh. Well, how did it become bigger? What did I do?" and the child will show you that you added blocks onto it. Then we'll start with the little one, saying, "Can you make this one as big as the other one? Can you make it even bigger?" And the child will add more blocks on to make it bigger. We do the same thing with a tower or with other forms and the child now is manipulating and adding on.

Then we do step three where we begin taking away things to make it smaller. "Let's see how small we can make it!" and do the same thing with trains or towers or other forms – it could be circles or squares. We can do it and the child can do it. Eventually we want the child to take the lead after he gets the idea, but we can show him first what we have in mind. Now the child is experimenting with adding and subtracting without calling it that or without counting.

Then the next step is to give the child some words to describe what's a "lot" and what's a "little." In other words, we give them the words for what they've just done. So now we give them numbers. Many children can already count, but they haven't necessarily put those words next to the object, so they haven't coordinated the "one" and the "two" with one block or two blocks or three blocks or four blocks. So now we add on numbers and we start with three and under – just one, two, and three. So, we say, "This is one, now we have two, now we have three." We do the same thing now, adding and subtracting, doing it with a train, doing it with a tower, etc., and we give it the name now, so the child has got "three" versus "one" or "two" versus "three" or "Which is the bigger" and "Which is the smaller?" You ask the child, "If you add one onto the two, how many do you have now?" Then he counts, "One, two, three." Then we ask, "If we take one away, how many are there now?" The child counts, "One, two." Now the child is adding or subtracting. Before we get to the adding and subtracting we want to make sure that the child recognizes in this pre-stage that regardless of whether it's in a tower form or the train form or another form, he's dealing with basically the same quantities of one object, like all blocks. The way we do that is have him experiment. We've got a tower of five blocks and a train of five blocks and the child says the train is more. So you say, "Okay take the tower and see if you can make a train out of it," and he'll see that he can do the same thing and we let him decide. Or maybe he takes his train and makes it into a tower and then he does it and he keeps doing it and playing with it until he "gets" it. So we don't progress to labeling the numbers until he "gets" the constancy of the sense of quantity, which is a form of conservation.

Then we progress to the numbers, starting with three and under; then we go to five; then we go to 10. We can even go to 20, all the while teaching him the name of the number. We can also script out the number so he can see it and so he knows what the numbers one through 10 look like. So he's seeing it, he's hearing it, he's saying it, and he's manipulating it, which will occur over many days or many weeks, depending how much time you spend on it, until the child "gets" it. Again, whenever you're not getting the interest of the child, pick something even more interesting to him, whether it's dinosaurs or cars or cookies or candies or whatever. There's nothing wrong with building an incentive where he gets to eat one of the cookies and figures out how many are left, or we get to bake some more cookies or find some more cookies. This is the way the child develops basic adding and subtracting. This is before we worry about writing down numbers or carrying over numbers of doing anything complicated; we want the child to have this basic sense of quantity.

Then we can do the same thing to make it a little more complicated once we've been through this with something very simple – using just our towers and our blocks and lining things up and making towers or circles or other forms or shapes the child may want to invent – then we can go to more complex perceptions where the child sees that things like clay can be stretched out in a snake or rolled up into a little ball and that it will be the same amount of clay. So we'll do the same thing with, "Which is more?" and let the child experiment with stringing it out into a snake. We can ask, "Can you make a ball out of it?" If he says the snake is more than the ball, then he makes the ball into a snake and matches it to the snake he already has. What does he conclude? We could do the same thing with volume, so we could put water in a short, fat glass and in a tall, thin glass and let the child transfer it from glass to glass, then asking him, "Which is more? Which is less?" So in this way the child begins getting a sense of the constancy, not just of blocks or cookies in a line or in a tower, but also of the amount of clay, the mass or weight of something, or the amount or the volume for liquids or fluids. Then, at a later point – not to get too complicated – we'll introduce this later after we get this concept across – that you can now weigh things like that ball of clay and the snake. You can put them on a scale and they'll weigh the same. So we can introduce the notion of which one is easier to pick up, which one is heavier, if the child has that word, and which one is lighter. We can ask, "Which one would give you more to drink?" if we're comparing fluid and volume and "Which one would give you less to drink?" We can show the child just like we can count the blocks by putting the fluid in a glass that has little marks on it that indicates its level and depending on whether we put a little bit or a lot in the same kind

of glass we can see whether it's at level five or level three or level two. We don't have to give the child, at this point, the proper terminology, i.e., whether we're talking about pints or quarts or milliliters, but we can give the child the notion that we can also apply our numbers to liquids or to things that go into different shapes and solids just to lay the foundations for later mathematical reasoning when we'll get into these things.

Also, as part of building this initial foundation for a sense of quantity, we can do the same thing once the child has the towers and the snakes and can apply numbers to those so the child has a sense of volume and a sense of mass or weight in different forms and different shapes without worrying too much about the numbers for that yet because those are more complicated, but just have that sense of that; then we can also get into the concepts that are going to be the foundation for multiplication and division and even percentages and decimals. Again, it's important for the child to have an experiential basis for this before he learns the formulas or tries to memorize any of the facts that go along with this. Here, again, it's even more important to pick something meaningful for the child, to take something for which he can use all his senses and can manipulate motorically. Remember, in all the steps I just gave you the child was invested in his blocks or his cookies; he was using his senses of sight and sound and touch; and he was motorically manipulating. We had the child doing it, so we satisfied our criteria for making this a full experience for the child. We know when we have a full experience the mind or brain is "cooking" on all channels at the same time, so it's very, very, very important.

Next we're going to give the child a sense of how we divide or multiply or add things up. Then we're going to take the same concept by taking our blocks or our cookies and we might put them in the form of a circle or make a little pie out of them a square pizza pie with the blocks or a rounded one with our circles or actually have the shape of a real pizza pie cut out of paper and cut into four pieces. Then we can say to the child, "Okay, here we have this thing and you want to get some and I want to get some and we both want to have about the same," and see if the child can figure out that if there are four pieces, then you get two blocks and I get two blocks. If the child can't do it and gives you one and him three, you can say, "Wait, wait! That's not the same – who has more?" The child looks, "I've got more. I want more!" "Okay, but just for fun, can we have the same, just for a minute, then you get to have more?" and see if the child can give us both the same. If he makes a mistake, keep having him go back to the drawing board until he can give us both the same amount. If you start off with just

two, it'll be easy for him to divide it. If you start out with four it's a little tougher, but at some point he'll get it. "No, no, no – wait, wait, wait. Who's got more?" He can already count, he can already do the one, two. So, eventually, by chance or luck he'll come on it and then we'll see if he can replicate it and do it with a very, very small number of something the child is invested in. It can be a little incentive or reward at the end if he gets it.

So now what we've done is we've divided and given the child a sense of dividing things into two. Then we can give him the word "half." "Okay, so of this whole thing, I have half and you have half. Half is when we divide it into two." Then you bring in a little teddy bear and a little elephant and now we want everyone to have the same amount. "We now have a pie of four, so you get the same, I get the same, Teddy gets the same and Elephant gets the same." Then the child experiments until each one gets one piece. So, okay, we've now given this pie to four people – we give him the word "division" – we divided it into four, so now he understands what "divide" means because he's experienced it, we've played with it. We play with that with cookies, with pizza pies, with other things until the child "gets" it – this is, just experientially; we're not concerned with the numbers here and we keep the numbers low, under five, until the child really, really gets it and can do it and play the game.

Then we can do the same thing with multiplication. Okay, Johnny can count already so how many are we going to need? "We've got me and you and we each want one piece. How many are we going to need?" Johnny can figure that one out probably pretty easily, "You need one and I need one, that's one plus one is two." Then you say, "Okay, now we each want two pieces. How many are we going to need?" He may say, "I can't figure it out!" You can say, "Well, give us each two and see how many there are when you do that. "There's one, two, three, four." Okay, so now we add Teddy and Elephant in. Each one wants one piece, then two, then three. What are we doing? Then we tell the child, "Well, there's actually a word for that – we're multiplying. Each one is getting two and we've got four people, it's called 'two times four,' so we've got two and we're giving it two four people and the way we describe that is 'two times four' or 'four times two.'" It'll be a little vague for the child – the word "times" will be a new word and the child won't quite understand it, but he gets the experience of multiplying – of giving a certain amount, like two – to four different people. Later on, say, "We'll show you how to figure that out real quickly." So we give the child a basis for multiplication and division.

Then while we're doing division we can give him other words, like, "Okay, we're dividing this pie in half" or "We're giving it to four different people so each one gets," and give him the word for that. "If each one is getting one piece of it and it's divided into four, that's called a quarter," and we show him what that looks like. But we don't ask him to worry about that and to be able to replicate that at the moment, only to recognize that the concept is there. We can even introduce the word "percentage" if the child is quick verbally, and give him fifty percent, 25 percent, 100 percent, because he may have heard those words already. The idea isn't to understand percentages or fractions at this point, because that will come later, but just to give the child the experiential basis for what that feels like, by dividing things into halves or thirds or quarters as we're playing with our pies and as we're playing with these things.

Now, again, I'm condensing this into one quick description, but this will be played out over many weeks. We don't go to these more advanced levels until we're got the early ones mastered – the simple addition and subtraction and simple understanding of the different aspects of quantity as constant, no matter what shape or form you put them in, and being able to also have the numbers in both visual form and auditory form to correspond to the actual quantity. Once we have that, we have the foundation in place and then we can add on the multiplication and the division, and then we can "flirt" a little bit with fractions and decimals in the ways I've just described using a multi-sensory, experientially based, highly emotionally meaningful context. Then the child has the foundations in place and can begin mastering the more formal parts of math and how the numbers go along with these and how you can write them down. Any one of the existing math systems that are experientially-based, that take things one step at a time, can be used. Later on when he's older, the child can even memorize his multiplication tables or memorize some of the simple math facts so he doesn't have to use his fingers. But it's better early on to use his fingers or to use blocks or manipulatables until he "gets" it.

Once the child has a strong foundation he can learn in many different modalities. Many of the curricula used at school will be fine, but many of the curricula don't have enough focus on these foundations and our focus here is on employing multi-sensory, motor-based experiences in an emotionally meaningful, human, interactive, fun context.

Some children have an easy time when they see the math – they see the numbers and they can learn to add and subtract and even divide and multiply, but they have a hard time with word problems because of a language problem or a sequencing

problem, and they have trouble transferring what they hear to what they see. Some children are just the opposite – they're better with word problems, but have a harder time when they have to line up their columns or just see visual symbols. Once we've mastered the basics and we've done it with the words and the visual images and the experiential basis of "more" or "less" that we've just described, then it's important that the child become very facile with both the verbal and the visual forms of math. Here, there's nothing like practice and having fun. The key elements for children is that they are invested in things that are fun and interesting; that they have a sense of mastery over it; and that they avoid things that are not fun, not interesting, and especially things that make him feel inadequate or as though they can't succeed.

Math needs to be individualized so the child experiences at least a 70% success rate. We always slow down the steps so there's slower progress, but we can use things that can be divided into three so the child develops mastery. If the child can't do it with five blocks, he can do it with three blocks. If he can't do it with three, he can do it with two blocks until he "gets" it. To help the child shift back and forth between the verbal and the visual we have to practice it. So, when you're teaching with word problems, say, "We're giving Johnny two and we're giving Sally two. How many do we need all together? Let's count: one, two, three, four." Then just write it down. Write Johnny's name and the number two under it and then write Sally's name and put the number two under hers, as well. Then write out, "2 + 2 = ?" and ask him, "How many do we need?" Have the child script out four or have him pick four from a multiple-choice option that includes the numbers 4, 5, 3, so he circles the right one.

Then have the child who can script already or has fine motor control actually write it out, "2 + 2 = 4." So now right from the very get-go we help the child become better. If the child's having more trouble with the verbal and it may be with verbal sequencing, so we make up little games where we give the child more verbal experience where he's in charge of it and we start very simple. Pick the child and his favorite animal or a few of his favorite animals, or the child and his sibling or his best friend – bring his best friend over – where they can each have two of something and we can have them act it out as the child describes it verbally so he can actually see it in real life. You can create a drama around it or create a pretend drama with the teddy bear and the dinosaur so that each one gets two blocks. We can bring the verbal alive visually and then have it represented on a piece of paper and vice-versa. We want to keep it going back and forth and we must be very patient. A child who doesn't sequence well may

need further work on the executive function or the sequencing capacities – motor planning and sequencing – that we talked about in the root system, and we're going to talk about a little bit more next time, too, as part of the branches. So from the get-go, let's have the child be very good at both visual and verbal and therefore math can strengthen language and language can strengthen math; also math can strengthen visual experiences and visual processing, and visual thinking exercises can strengthen the math. Again, too, some of the children who do math will have to go back to that root system and work on their visual-spatial processing. If you go back and re-read that section or re-listen to it you'll see that we're covering a lot of the same territory, a lot of the same basic underlying fundamentals.

Now next time we're going to talk about a couple of things – we're going to talk a little bit more about executive functioning and sequencing and planning and following directions, which is part of the branches, as well as the root system; and we're going to talk about how you help the child cope with the emotional parts of learning when he gets discouraged and overwhelmed or gets anxious or feels he can't do something or he gets avoidant. If there's not time the next time, the time after we'll talk about how we help whole schools become learning environments because we know there is a lot of concern now that schools are not adequately educating children and are not being successful. Some of that is because children are coming less ready for school in some areas, but we have to help all children – those who are less ready and those who are more ready – because the job of education is to start where the child is. So if we expect too much we can't lower our expectations, but we have to start at the beginning. So we'll cover how to help schools get "cooking" as a whole organization – how to involve the parents, how to get teachers, optimally, feeling motivated and getting the whole community to become a learning community. So, those will be our remaining components of this series on learning and that will take us one or two more segments to do.

Thank you for participating in today's session and we will speak to you again soon.